Exercise 16

For the following exercises, determine whether the relation represents y as a function of x.

$$x = \sqrt{1 - y^2}$$

Solution

Try to solve the given equation for y. Start by squaring both sides to remove the square root on the right side.

$$x^2 = 1 - y^2$$

Isolate the term with y.

$$y^2 = 1 - x^2$$

Take the square root of both sides.

$$\sqrt{y^2} = \sqrt{1 - x^2}$$

Because there's an even power under an even root, and the result is to an odd power (y^1), an absolute value sign is needed.

$$|y| = \sqrt{1 - x^2}$$

Remove the absolute value by placing $\pm$ on the right side.

$$y = \pm\sqrt{1 - x^2}$$

The relation $x = \sqrt{1 - y^2}$ is not a function because for every input x, there are two outputs given by $y = \sqrt{1 - x^2}$ and $y = -\sqrt{1 - x^2}$. This is reflected in the graph by the fact that there are vertical lines that pass through the curve more than once.